# Physics Notes

BY

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Class:10+1
Unit: V
Topic: Motion of System of Particles and Rigid Body

*__SYLLABUS__*: UNIT-V

Centre of mass of a two particle system, momentum conversion and centre of mass motion. Centre of mass of a rigid body; centre of mass of uniform rod.
Vector product of vectors; moment of a force, torque, angular momentum, conservation of angular momentum with some examples.
Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions; moment of inertia, radius of gyration.
Values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorem and their application.

**Q.1. Prove**

**b)** **w** $\rightarrow$ **Angular Velocity at anu time**

$\boldsymbol{w_0}$ $\rightarrow$ **Initial Angular Velocity**

**α** $\rightarrow$ **Angular Acceleration**

**c)** **s = ut +** $\boldsymbol{\frac{1}{2} at^2}$ **and θ =** $\boldsymbol{w_0}$ **t +** $\boldsymbol{\frac{1}{2} \alpha\, t^2}$

**d)** $\boldsymbol{v^2}$ **-** $\boldsymbol{u^2}$ **= 2as and** $\boldsymbol{w^2}$ **-** $\boldsymbol{{w_0}^2}$ **= 2 α θ**

Ans.

**LINEAR**

a)

| | |
|---|---|
| v | = v + at |
| acc | = a |
| $\frac{dv}{dt}$ | = a |
| $dv$ | = a . $dt$ |

Integrate both sides

| | |
|---|---|
| $\int dv$ | = $\int$ a . $dt$ |
| $\lvert v \rvert_{v=u}^{v=v}$ | = a $\lvert t \rvert_{t=0}^{t=t}$ |
| $v - u$ | = a(t – 0) |

$$\boxed{v = u + at}$$

**ANGULAR**

| | |
|---|---|
| w | = $w_0$ + α t |
| ang. acc | = α |
| $\frac{dw}{dt}$ | = α |
| $dw$ | = α . $dt$ |

Integrate both sides

| | |
|---|---|
| $\int dw$ | = $\int$ α . $dt$ |
| $\lvert w \rvert_{w=w_0}^{w=w}$ | = α $\lvert t \rvert_{t=0}^{t=t}$ |
| $w - w_0$ | = α (t – 0) |

$$\boxed{w = w_0 + \alpha t}$$

**LINEAR**

b)

| | |
|---|---|
| s | = ut + $\frac{1}{2} at^2$ |
| $v - u$ | = at |
| $v$ | = $u$ + at |
| $\frac{ds}{dt}$ | = $u$ + at |
| $ds$ | = $u\ dt + at\ dt$ |

Integrate both sides

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=t} u.dt + \int_{t=0}^{t=t} at.dt$$

| | |
|---|---|
| s-0 | = u $\lvert t - 0 \rvert$ + a $\left\lvert \frac{t^{1+1}}{2} - 0 \right\rvert$ |
| s | = ut + $\frac{at^2}{2}$ -0 |

$$\boxed{s = ut + \frac{1}{2} at^2}$$

**ANGULAR**

| | |
|---|---|
| θ | = $w_0$t + $\frac{1}{2}$ α $t^2$ |
| w-$w_0$ | = α t |
| w | = $w_0$ + α t |
| $\frac{d\theta}{dt}$ | = $w_0$ + α t |
| $d\theta$ | = $dt\ w_0$ + α t $dt$ |

Integrate both sides

| | |
|---|---|
| $\theta$-0 | = $w_0 \lvert t - 0 \rvert$ + α $\left\lvert \frac{t^2}{2} - 0 \right\rvert$ |
| $\theta$ | = $w_0$t + $\frac{\alpha t^2}{2}$ |

$$\boxed{\theta = w_0 t + \frac{1}{2} \alpha t^2}$$

| LINEAR | | ANGULAR | |
|---|---|---|---|
| c) $v^2$-$u^2$ | = 2 as | w$^2$-$w_0{}^2$ | = 2 α $\theta$ |
| acc. | = a | $\frac{dw}{dt}$ | = α |
| $\frac{dv}{dt}$ | = a | $\frac{dw\ d\theta}{dt\ d\theta}$ | = α |
| $\frac{dv\ ds}{dt\ ds}$ | = a | $dw$ w | = α $d\theta$ |
| $dv\ v$ | = a $ds$ | $\int w.dw$ | = $\int$ α$.d\theta$ (Integrating) |
| $\int v.dv$ | = $\int a.ds$ (Integrating) | $\left\|\frac{w^2}{2}\right\|_{w_0}^{w}$ | = α. $\|\theta\|_{\theta=0}^{\theta=\theta}$ |
| $\left\|\frac{v^{1+1}}{2}\right\|_{u}^{v}$ | = a $\|s\|_{0}^{s}$ | $\left\|\frac{w^2}{2}\right\|_{w_0}^{w}$ | = α $\|\theta - 0\|$ |
| $\left\|\frac{v^2}{2}\right\|_{u}^{v}$ | = a $\|s - 0\|$ | $\frac{w^2}{2}$ - $\frac{w_0{}^2}{2}$ | = α $\theta$ |
| $\boxed{v^2\text{-}u^2 = 2\text{ as}}$ | | $\boxed{w^2\text{-}\ w_0{}^2 = 2\ \alpha\ \theta}$ | |

**Q2. Cross product of two vectors? Right hand rule for direction? Geometrical interpretation? Properties? Special cases?**

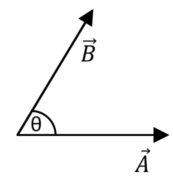
Ans.

a) **<u>Vector Product</u>**:

$\vec{C} = \vec{A} \times \vec{B}$

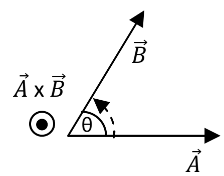
$|\vec{C}| = A\ B \sin\theta$

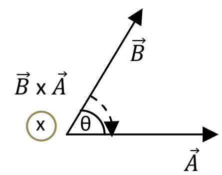
Cross product of two vectors $\vec{A}$ and $\vec{B}$ is product of magnitude of $\vec{A}$, $\vec{B}$ and sine of angle between the two vector.




b) **<u>Right Hand Rule</u>**:

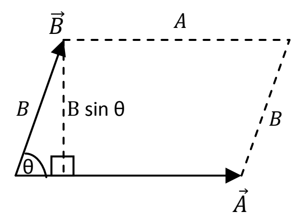
If we roll our fingers of right hand from $\vec{A}$ and $\vec{B}$, thumb point in director of $\vec{A} \times \vec{B}$.




c) **<u>Geometrical Interpretation</u>**:

$|\vec{A} \times \vec{B}| = A\ B \sin\theta$
$= A\ (B \sin\theta)$
= Area of //gm of sides A and B




d) **<u>Properties</u>**:

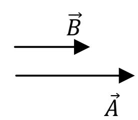
1. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
2. Distributive $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
3. Associative $(\vec{A} + \vec{B})\ (\vec{C} + \vec{D}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{D} + \vec{B} \times \vec{C} + \vec{B} \times \vec{D}$

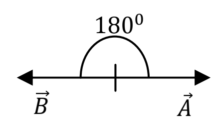
e) **<u>Special Cases</u>**:

i) $\theta = 0^0$ or $180^0$

$|\vec{A} \times \vec{B}| = A\ B \sin\theta$
$= 0$

$\hat{\imath} \times \hat{\imath} = 0,\quad \hat{\jmath} \times \hat{\jmath} = 0,\quad \hat{k} \times \hat{k} = 0$

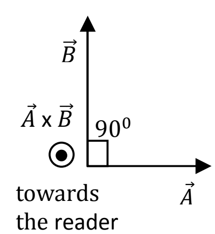



ii) $\vec{A} \perp \vec{B}$

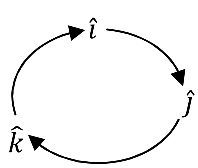
$|\vec{A} \times \vec{B}| = A\ B \sin 90$
$= AB$

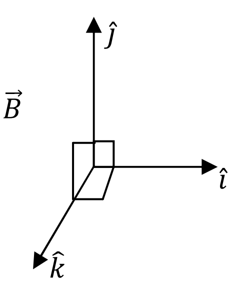
Direction: $\vec{A} \times \vec{B}$ is ⊙
i.e. towards the reader
$(\vec{A} \times \vec{B}) \perp \vec{A}$
$(\vec{A} \times \vec{B}) \perp \vec{B}$
$(\vec{A} \times \vec{B}) \perp$ to plane containing $\vec{A}$ and $\vec{B}$

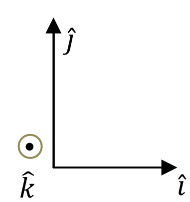



$\hat{\imath} \times \hat{\jmath} = \hat{k}$
$\hat{\jmath} \times \hat{k} = \hat{\imath}$
$\hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$
$\hat{k} \times \hat{\jmath} = -\hat{\imath}$
$\hat{\imath} \times \hat{k} = -\hat{\jmath}$



**Q3. What is a system? Explain "internal force" and "external force" by example.**

Ans. **<u>System</u>**:

A collection of any number of particles interacting with one another is called a *system*.

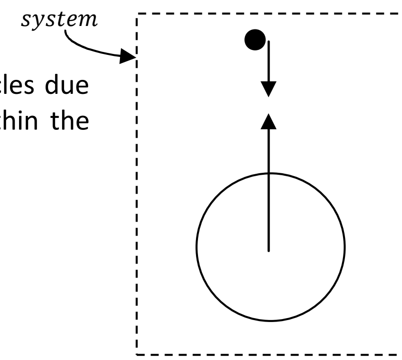
**<u>Internal Force</u>**:

Internal Force is force developed between particles due to mutual interaction and source of force is within the system.

<u>Example</u>:

Force between Earth and Ball

here $\overrightarrow{F_{AB}} = -\overrightarrow{F_{BA}}$




$$F = \frac{GM_m}{r^2}$$

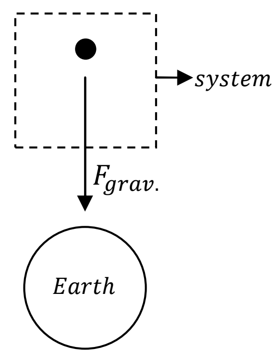
**<u>External Force</u>**:

Force having its source (cause) outside the system is called *external force*.

<u>Example</u>:

Force on ball (taking only ball as system) is due to gravity.

here $F_{grav.}$ is external force.



**Q4. Discuss three types of motions i.e.**

**a) Pure Translational Motion**

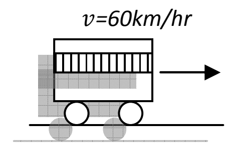
**b) Pure Rotational Motion**

**c) Combination of Translational & Rotational Motion.**

Ans.

a) **Translational Motion**:

In such a motion every particle of the body has the same velocity at a particular instant of time.

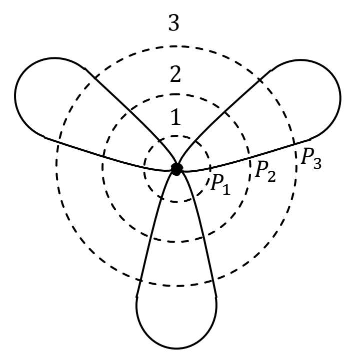



Example:

A bus moves with speed 60km/hr. Then all the passengers sitting in the bus also move with the same speed of 60km/hr. This is because the bus is rigid and in this type of motion, center of mass moves with same velocity i.e. 60km/hr.

b) **Rotational Motion**:

In such a motion, a rigid body rotates about a fixed axis. Every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre of mass on the axis.




Example:

Blades of a fan move in rotatory motion.

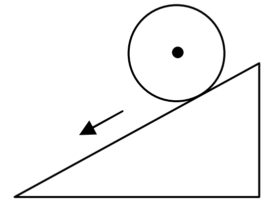
Particle $P_1$ describes a circle.
Particle $P_2$ describes a circle.
Particle $P_3$ describes a circle.

The centre of three circles lies on the same axis.

- The angular velocity $w$ of all the particle is same irrespective of their distances from the axis of rotation (radius).
- However, their linear velocities are different $(v = rw)$ depending on their distance ($r$) from the axis.

c) **Combination of Translational & Rotational Motion**:

When a cylinder rolls down an inclined plane, its motion is a combination of rotation about a fixed axis and translation.



Example:

The wheels of a car or bus are also an example of this type of motion.

**Q5. What is Centre of Mass. Write expression for**

**a) Position of centre of Mass.**

**b) Velocity of centre of Mass.**

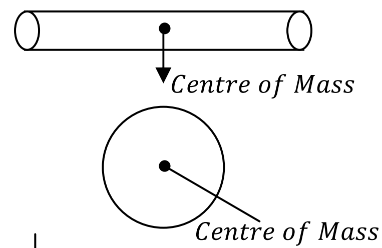
**c) Acceleration of centre of Mass.**

Ans. **Centre of Mass**:

Centre of Mass is a points at which the entire mass of the body/system is supposed to be concentrated.

Example 1. A rod

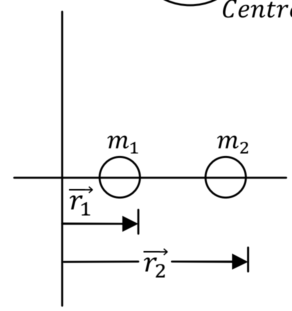
Example 2. Disc, Ring etc.




a) **Position of Centre of Mass**:

$$\overrightarrow{r_{c.m}} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2} \quad \text{(for 2 particles)}$$

$$\overrightarrow{r_{c.m}} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2} + m_3 \overrightarrow{r_3}}{m_1 + m_2 + m_3} \quad \text{(for 3 particles)}$$




b) **Velocity of Centre of Mass**:

$$\overrightarrow{V_{c.m}} = \frac{d}{dt}\left(\overrightarrow{r_{c.m}}\right)$$

$$= \frac{d}{dt}\left(\frac{m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2}}{m_1 + m_2}\right)$$

$$= \frac{m_1\left(\frac{d\overrightarrow{r_1}}{dt}\right) + m_2\left(\frac{d\overrightarrow{r_2}}{dt}\right)}{m_1 + m_2}$$

$$\boxed{\overrightarrow{V_{c.m}} = \frac{m_1 \overrightarrow{V_1} + m_2 \overrightarrow{V_2}}{m_1 + m_2}}$$

For n particles:

$$\overrightarrow{V_{c.m}} = \frac{m_1 \overrightarrow{V_1} + m_2 \overrightarrow{V_2} + \text{-----}\ m_n \overrightarrow{V_n}}{m_1 + m_2 + \text{-----}\ m_n}$$

c) **Acceleration of Centre of Mass**:

$$\overrightarrow{a_{c.m}} = \frac{d}{dt}\left(\overrightarrow{V_{c.m}}\right)$$

$$= \frac{d}{dt}\left(\frac{m_1\overrightarrow{V_1} + m_2\overrightarrow{V_2}}{m_1 + m_2}\right)$$

$$= \frac{m_1\left(\frac{d\overrightarrow{V_1}}{dt}\right) + m_2\left(\frac{d\overrightarrow{V_2}}{dt}\right)}{m_1 + m_2}$$

$$\boxed{\overrightarrow{a_{c.m}} = \frac{m_1 \overrightarrow{a_1} + m_2 \overrightarrow{a_2}}{m_1 + m_2}}$$

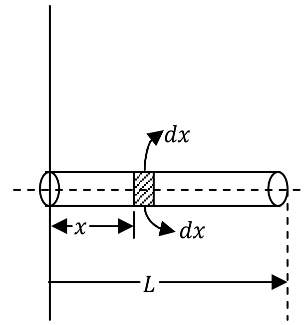
For n particles:

$$\overrightarrow{a_{c.m}} = \frac{m_1 \overrightarrow{a_1} + m_2 \overrightarrow{a_2} + \text{-----}\ m_n \overrightarrow{a_n}}{m_1 + m_2 + \text{-----}\ m_n}$$

**Q6. Derive an expression for centre of Mass of a rod of length *l*.**

Ans. $X_{c.m} = \dfrac{\int dm.x}{\int dm}$

$= \dfrac{\int \left(\frac{M}{L} dx\right) x}{\int dm}$ $\left(\begin{array}{r} M \rightarrow Mass\ of\ rod \\ of\ length\ L \end{array}\right)$




mass per unit length
$\lambda = \frac{M}{L}$

$= \dfrac{\frac{\cancel{M}}{L} \int x\,.dx}{\cancel{M}}$

$= \frac{1}{L} \int x\,.\,dx$ $\left[\int x^n\,.\,dx = \frac{x^{n+1}}{n+1}\right]$

$= \frac{1}{L} \left| \frac{x^{1+1}}{1+1} \right|_{x=0}^{x=L}$

$= \frac{1}{L} \left| \frac{x^2}{2} \right|_{x=0}^{x=L}$

$= \frac{1}{L} \times \frac{1}{2} (L^2 - 0^2)$

$= \frac{1}{L} \times \frac{1}{2} L^2$

$$\boxed{X_{c.m} = \frac{L}{2}}$$

Centre of mass of a rod of uniformly distributed mass lies at centre.

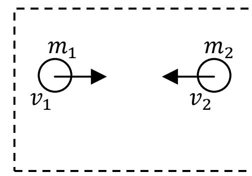
**Q7. a) Discuss velocity of centre of Mass, when $F_{external}$ on the system is zero.**

**b) Examples of motion of centre of Mass for two/more than two particle system.**

Ans.a) Momentum of Particle 1, $\overrightarrow{P_1} = m_1 \overrightarrow{v_1}$

Momentum of Particle 2, $\overrightarrow{P_2} = m_2 \overrightarrow{v_2}$

Momentum of Particle n, $\overrightarrow{P_n} = m_n \overrightarrow{v_n}$




Total Momentum of the system, $\vec{P} = \overrightarrow{P_1} + \overrightarrow{P_2}$ --------- $+ \overrightarrow{P_n}$

$\vec{P} = \text{M}\, \overrightarrow{V_{C.M}}$

$\vec{F}_{external} = \frac{d\vec{P}}{dt}$

If $\vec{F}_{external} = 0$

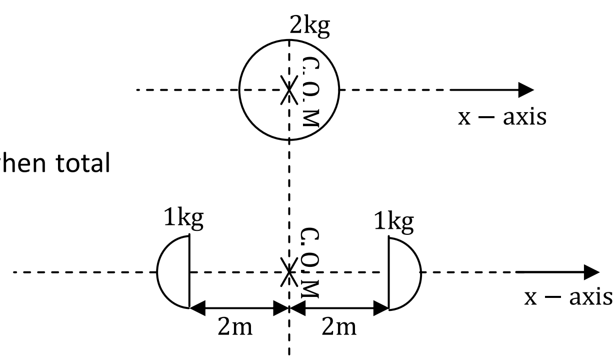
$\frac{d\vec{P}}{dt} = 0$

$\vec{P}$ is constant

M $\overrightarrow{V_{C.M}}$ is constant

$$\boxed{\overrightarrow{V_{C.M}} \rightarrow \text{constant}}$$

Velocity of centre of mass remains constant when total external force on the system is zero.

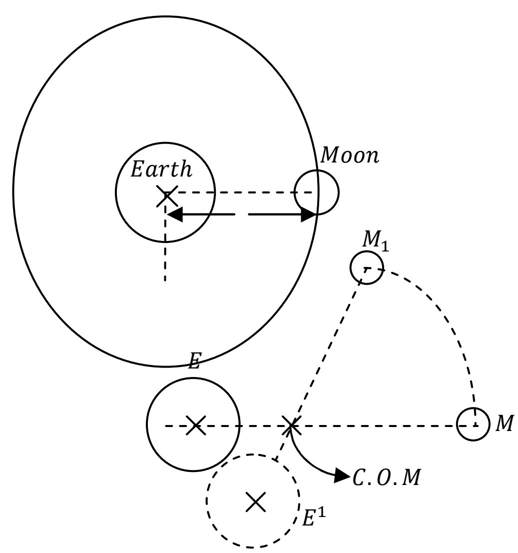



b) **<u>Examples of Motion of Centre of Mass</u>**:

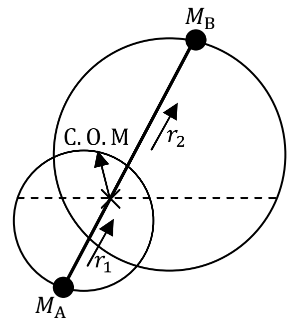
i) **Explosion of Bomb**:
Velocity of Centre of Mass before and after collision is *zero*, because $F_{external} = 0$

ii) **Wrong Statement**:
Moon revolves around the Earth in a circle with its centre coinciding with the centre of the Earth.

**Correct Statement**:
This reason rules out the possibility of Earth and Moon colliding against each other under the internal gravitational forces.
Both Earth and Moon revolve around their common centre of mass. They are always on the opposite sides of the common centre of mass.
As mass of Earth is very large in comparison to mass of Moon, centre of mass lies close to the Earth.



iii) **Binary Stars**:
Two stars which move in their elliptical orbits around a common centre of mass are said to form *Binary Stars*.




$$\overrightarrow{r_{cm}} = \frac{M_A \vec{r_1} + M_B \vec{r_2}}{M_A + M_B}$$

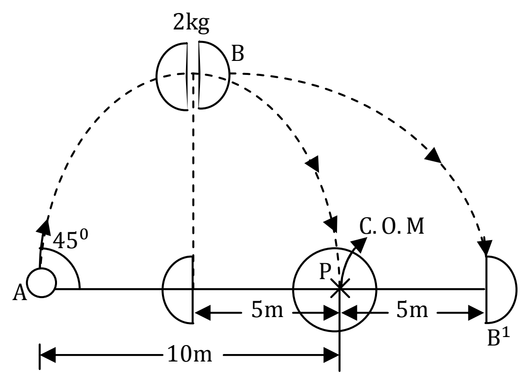
As centre of mass coincides with the point about which the two stars orbit, therefore $\overrightarrow{r_{c.m}} = 0$

$$M_A \vec{r_1} + M_B \vec{r_2} = 0$$

$$\frac{M_A}{M_B} = -\frac{r_2}{r_1}$$

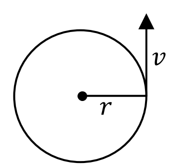
*OR* $\boxed{M_A \vec{r_1} = -M_B \vec{r_2}}$

iv) When a bomb is fired at an angle θ as shown in figure, the bomb reaches point *P* in case of "*no explosion*".
If the bomb explodes at highest position, centre of mass will follow the same trajectory as in previous case i.e. "*no explosion*".




**Q8. Write and explain vector relation between $\vec{v}$ and $\vec{w}$ ?**

Ans. **Scalar Relation**:



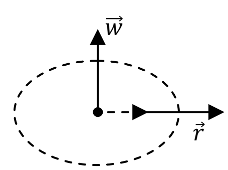

$$v = rw$$

$$v = \frac{total\ distance}{total\ time}$$

$$= \frac{2\pi r}{T}$$

$$= \left(\frac{2\pi}{T}\right) r$$

$$\boxed{v = rw}$$




**Vector Relation**:

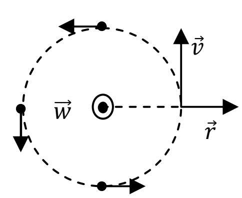
$\vec{w}$ is axial vector

$\vec{r}$ is radial vector

$\vec{v}$ is tangential vector

$$\boxed{\vec{v} = \vec{w} \times \vec{r}}$$

$$\boxed{\text{not } \vec{v} = \vec{r} \times \vec{w}}$$




*Towards reader*

**Q9. Write and explain vector relation $\vec{\alpha}$ and $\vec{a}$ ? ($\vec{\alpha}$ angular acc., $\vec{a}$ linear acc.).**

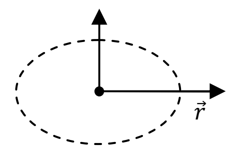
Ans. **<u>Scalar Relation</u>**: $\boxed{a = \alpha r}$

$$a = \frac{total\ velocity}{time}$$

$$= \frac{rw}{t}$$

$$= r\left(\frac{w}{t}\right)$$

$$\boxed{a = r\,\alpha}$$




**<u>Vector Relation</u>**:

$\vec{\alpha}$ is angular acceleration

$\vec{a}$ is acceleration

$\vec{r}$ is radial vector

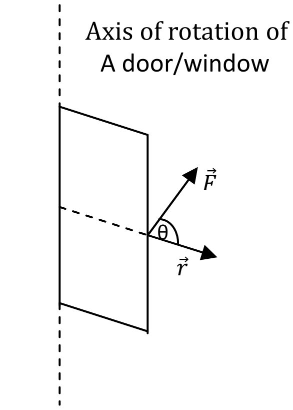
$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$$

**Q10. Moment of Force or Torque?**

Ans. **<u>TORQUE</u>**:

The moment of force on torque is the turning effect of the force about the fixed point/axis.

It is give by product of magnitude of $r$ and magnitude of *F* and the sine of angle between them.




$\vec{\tau} \quad = \vec{r} \times \vec{F}$

$\vec{\tau} \quad = r . F . \sin\theta$

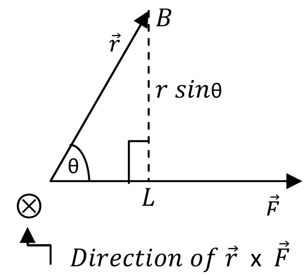
$= F(r \sin\theta)$

$=$ (Force) ($\perp$ distance BL)

Torque is axial vector

Direction of $\vec{r} \times \vec{F}$

$\perp$ to the plain containing $\vec{r}$ and $\vec{F}$




**<u>Analogy</u>**:

| **LINEAR** | **ANGULAR** |
|---|---|
| 1. Force, *F* | 1. Torque, τ |
| 2. F $= \frac{d}{dt}$ (Linear Momentum) | 2. τ $= \frac{d}{dt}$ (Angular Momentum) |
| $\boxed{F = \frac{d}{dt}(P)}$ | $\boxed{\tau = \frac{d}{dt}(L)}$ |
| 3. F $= \frac{d}{dt}(m\,v)$ | 3. τ $= \frac{d}{dt}(I.\omega)$ |
| $= m . \frac{dv}{dt}$ | Where I → moment of inertia $\simeq m\,r^2$ |
| $= ma$ | τ $= \frac{d}{dt}(I.\omega)$ |
| | $= I . \left(\frac{dw}{dt}\right)$ |
| | $= I\,\alpha$ |
| | (α → angular acceleration) |

**Q11. Write an expression for power in rotational motion?**

Ans.

**LINEAR**

Power, $P = \frac{work}{time}$

$P = \frac{dw}{dt}$

$P = \frac{d(\vec{F}\ \vec{s})}{dt}$

$P = \vec{F}\left(\frac{\vec{ds}}{dt}\right)$

$$\boxed{P = \vec{F}\ .\ \vec{v}}$$

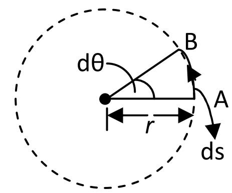
**ROTATIONAL**

$P = \frac{work}{time}$

$= \frac{F\ ds}{dt}$

$= (F\ r)\frac{d\theta}{dt}$

$= \tau\ (\omega)$

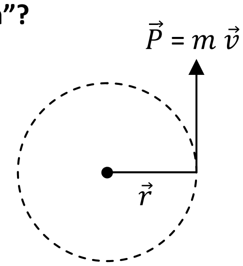


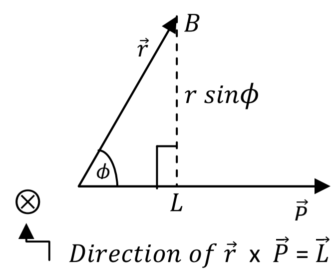

$$\boxed{P = \tau\ \omega}$$

Power associated with torque is given by product of torque and angular velocity of the body about the axis of rotation.

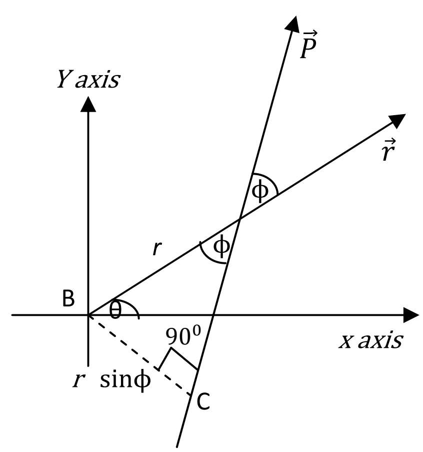
**Q12. Write an expression for “angular momentum”?**

Ans. <u>**Angular Momentum**</u>:




$\vec{L} = \vec{r}\ .\ \vec{p}$




$|\vec{r} \times \vec{p}| = r\ p\ sin\phi$

$$\boxed{L = r\ p\ sin\phi}$$

$= p\ (r\ sin\phi)$

$= p\ (\perp distance\ BC)$